

Improvement in the signal amplitude and bandwidth of an optical atomic magnetometer via alignment-to-orientation conversion: supplement

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1. THE CALCULATION FOR THE OPTICAL ROTATION AT THE $F = 1 \rightarrow F' = 0$ TRANSITION

Consider the case of a closed transition $F = 1 \rightarrow F' = 0$. The analytic solution of optical rotation angle of transition was derived from the Liouville equation via the ADM package [1]. The rotation angle φ can be simplified to a superimposed form comprising two dispersion curves as follows:

$$\frac{d\varphi}{dz} = -\frac{2\pi\Gamma}{24} \left(k_+ \frac{b_+ \Omega_L}{\Omega_L^2 + b_+^2} + k_- \frac{b_- \Omega_L}{\Omega_L^2 + b_-^2} \right). \quad (\text{S1})$$

where

$$k_{\pm} = a_{\pm}/b_{\pm}, \quad (\text{S2})$$

$$a_{\pm} = 1 \pm 1/\sqrt{1 - f(\tilde{\Omega}_R^2)\tilde{\Omega}_R^2}, \quad (\text{S3})$$

$$b_{\pm} = \frac{1}{2} \sqrt{\gamma^2 + ((\Gamma + 2\gamma)^2 - \gamma^2) \frac{1}{2} \left(1 \pm \sqrt{1 + A_3 \tilde{\Omega}_R^2 + A_4 \tilde{\Omega}_R^4} \right) + A_2 \tilde{\Omega}_R^2}. \quad (\text{S4})$$

The coefficients can be further classified as Ω_R (Eq. (S5) and Eq. (S6)) and without Ω_R (Eq. (S7) – Eq. (S10)) as follows:

$$f(\tilde{\Omega}_R^2) = \frac{4(1 + A_1 \tilde{\Omega}_R^2)}{((\Gamma + 2\gamma)(1 + A_1 \tilde{\Omega}_R^2) - \gamma)^2}, \quad (\text{S5})$$

$$\tilde{\Omega}_R^2 = \frac{\Omega_R^2}{3\gamma(\Gamma + \gamma)^2(\Gamma + 3\gamma)}, \quad (\text{S6})$$

$$A_1 = (\Gamma + 9\gamma)(\Gamma + \gamma), \quad (\text{S7})$$

$$A_2 = (\Gamma^2 + 5\Gamma\gamma + 12\gamma^2)(\Gamma + \gamma)(\Gamma + 3\gamma)/2, \quad (\text{S8})$$

$$A_3 = 2\Gamma(\Gamma + 5\gamma), \quad (\text{S9})$$

$$A_4 = \Gamma(\Gamma^3 + 10\Gamma^2\gamma + \Gamma\gamma^2 - 72\gamma^3). \quad (\text{S10})$$

Where γ is the relaxation rate of depolarization; Γ is the spontaneous emission rate; Ω_L is the Larmor frequency; and Ω_R is the Rabi frequency, which depends on the laser intensity.

The effect of power on the polarization angle was analyzed based on Eq. (S1). In these expressions, k_{\pm} and $2b_{\pm}$ represent the amplitudes and linewidths of the two dispersion curves, respectively. For the lower laser power limit ($\Omega_R^2 \rightarrow 0$), it can be inferred from Eq. (S2) – Eq. (S10) that $k_+ = 4(\Gamma + 2\gamma)^{-1}$, $2b_+ = \Gamma + 2\gamma \approx \Gamma$ (for most case, $\Gamma \gg \gamma$), $k_- = 0$, and $2b_- = \gamma$. The relationship between linewidth and relaxation indicate that the curves $k_+(b_+ \Omega_L / (\Omega_L^2 + b_+^2))$ and $k_-(b_- \Omega_L / (\Omega_L^2 + b_-^2))$ were classified as contributing to the linear Faraday effect and NMOR effects (coherence effect domain), respectively. Evidently, the rotation signs of the two curves are opposite from Eq. (S1).

The two components of the multidispersion curves were analyzed separately in Fig. S1. Figure S1(a) shows the amplitude of the dispersion curve as a function of the optical pumping saturation parameter $\kappa = \Omega_R^2 / (\Gamma\gamma)$. For the linear Faraday effect curve, the amplitude decreases

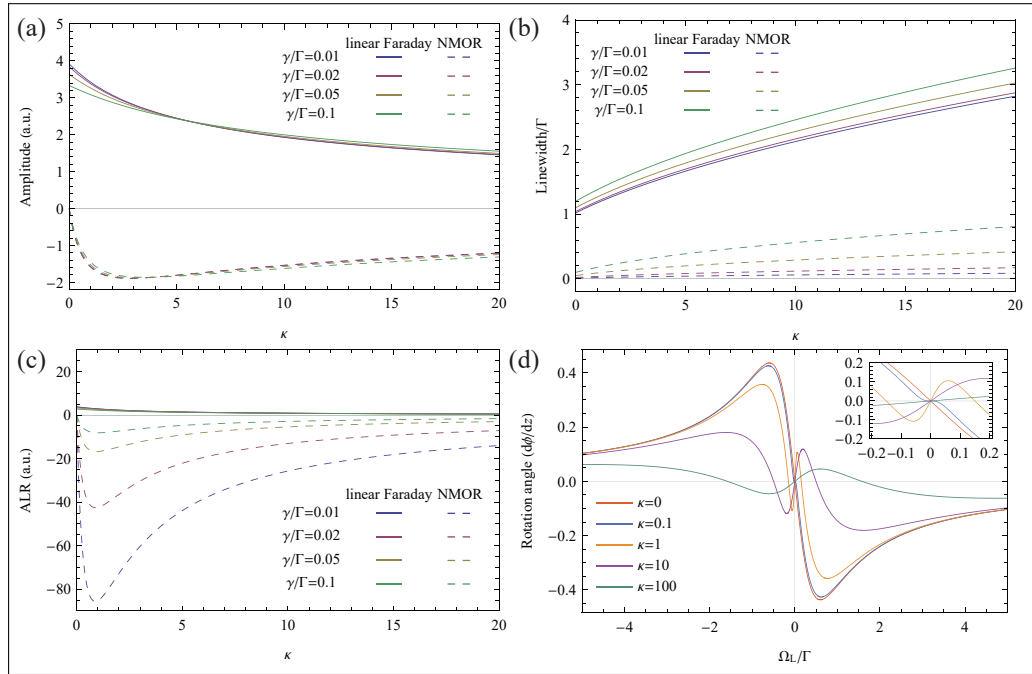


Fig. S1. Simulation results for the transition of $F = 1 \rightarrow F' = 0$. (a) Amplitudes of the dispersion curves as functions of κ . (b) Linewidths of the dispersion curves as functions of κ . (c) ALRs of the dispersion curves as functions of κ . (d) The magneto-optical signal at different κ where $\gamma = 0.1\Gamma$. The inset shows the rotation angle near zero magnetic field. With the increase in the laser power, the contribution of rotation angle changes from being linearly Faraday effect dominated to NMOR-effect dominated.

monotonically with κ . For the NMOR effect curve, the amplitude increases at the beginning and reaches maximum at the $\kappa \approx 1$, then decreases as the κ is increased. The linewidth of the two dispersion curves are shown in Fig. S1(b). The curves owing to the NMOR effect have larger ALRs than the linear Faraday effect curves, as shown in Fig. S1(c). Figure S1(d) shows the relation between the optical rotation angle and the magnetic field at different κ . The inset shows the rotation angle near zero magnetic field. A reversal of optical rotation sign can be found when comparing the curves of $\kappa = 0$ and $\kappa = 1$. This reversal reflects the change of the main contribution of rotation angle from linear Faraday effect to NMOR effect. Then, there is no reversal of rotation sign in the κ range from 1 to 100, which is consistent with the results of previous work [2].

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